The integral is evaluated as follows:

$$\begin{aligned}
\int \frac{dx}{1+\cos x} &= \int \frac{1}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}\,dx && \text{Multiply by 1.}\\
&= \int \frac{1-\cos x}{1-\cos^2 x}\,dx && \text{Simplify.}\\
&= \int \frac{1-\cos x}{\sin^2 x}\,dx && 1-\cos^2 x = \sin^2 x\\
&= \int \frac{1}{\sin^2 x}\,dx - \int \frac{\cos x}{\sin^2 x}\,dx && \text{Split up the fraction.}\\
&= \int \csc^2 x\,dx - \int \csc x \cot x\,dx && \csc x = \frac{1}{\sin x},\ \cot x = \frac{\cos x}{\sin x}\\
&= -\cot x + \csc x + C. && \text{Integrate using Table 7.1.}
\end{aligned}$$

Related Exercises 37–40 ◂

The techniques illustrated in this section are designed to transform or simplify an integrand before you apply a specific method. In fact, these ideas may help you recognize the best method to use. Keep them in mind as you learn new integration methods and improve your integration skills.

SECTION 7.1 EXERCISES

Review Questions

1. What change of variables would you use for the integral $\int (4-7x)^{-6}\,dx$?
2. Before integrating, how would you rewrite the integrand of $\int (x^4+2)^2\,dx$?
3. What trigonometric identity is useful in evaluating $\int \sin^2 x\,dx$?
4. Describe a first step in integrating $\int \frac{x^3-2x+4}{x-1}\,dx$.
5. Describe a first step in integrating $\int \frac{10}{x^2-4x+5}\,dx$.
6. Describe a first step in integrating $\int \frac{x^{10}-2x^4+10x^2+1}{3x^3}\,dx$.

Basic Skills

7–14. Substitution Review *Evaluate the following integrals.*

7. $\int \frac{dx}{(3-5x)^4}$
8. $\int (9x-2)^{-3}\,dx$
9. $\int_0^{3\pi/8} \sin\left(2x-\frac{\pi}{4}\right)dx$
10. $\int e^{3-4x}\,dx$
11. $\int \frac{\ln 2x}{x}\,dx$
12. $\int_{-5}^{0} \frac{dx}{\sqrt{4-x}}$
13. $\int \frac{e^x}{e^x+1}\,dx$
14. $\int \frac{e^{2\sqrt{y}+1}}{\sqrt{y}}\,dy$

15–22. Subtle substitutions *Evaluate the following integrals.*

15. $\int \frac{e^x}{e^x-2e^{-x}}\,dx$
16. $\int \frac{e^{2z}}{e^{2z}-4e^{-z}}\,dz$
17. $\int_1^{e^2} \frac{\ln^2(x^2)}{x}\,dx$
18. $\int \frac{\sin^3 x}{\cos^5 x}\,dx$
19. $\int \frac{\cos^4 x}{\sin^6 x}\,dx$
20. $\int_0^2 \frac{x(3x+2)}{\sqrt{x^3+x^2+4}}\,dx$
21. $\int \frac{dx}{x^{-1}+1}$
22. $\int \frac{dy}{y^{-1}+y^{-3}}$

23–28. Splitting fractions *Evaluate the following integrals.*

23. $\int \frac{x+2}{x^2+4}\,dx$
24. $\int_4^9 \frac{x^{5/2}-x^{1/2}}{x^{3/2}}\,dx$
25. $\int \frac{\sin t+\tan t}{\cos^2 t}\,dt$
26. $\int \frac{4+e^{-2x}}{e^{3x}}\,dx$
27. $\int \frac{2-3x}{\sqrt{1-x^2}}\,dx$
28. $\int \frac{3x+1}{\sqrt{4-x^2}}\,dx$

29–32. Division with rational functions *Evaluate the following integrals.*

29. $\int \frac{x+2}{x+4}\,dx$
30. $\int_2^4 \frac{x^2+2}{x-1}\,dx$
31. $\int \frac{t^3-2}{t+1}\,dt$
32. $\int \frac{6-x^4}{x^2+4}\,dx$

33–36. Completing the square *Evaluate the following integrals.*

33. $\displaystyle\int \frac{dx}{x^2 - 2x + 10}$

34. $\displaystyle\int_0^2 \frac{x}{x^2 + 4x + 8}\,dx$

35. $\displaystyle\int \frac{d\theta}{\sqrt{27 - 6\theta - \theta^2}}$

36. $\displaystyle\int \frac{x}{x^4 + 2x^2 + 1}\,dx$

37–40. Multiply by 1 *Evaluate the following integrals.*

37. $\displaystyle\int \frac{d\theta}{1 + \sin\theta}$

38. $\displaystyle\int \frac{1 - x}{1 - \sqrt{x}}\,dx$

39. $\displaystyle\int \frac{dx}{\sec x - 1}$

40. $\displaystyle\int \frac{d\theta}{1 - \csc\theta}$

Further Explorations

41. **Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample.

a. $\displaystyle\int \frac{3}{x^2 + 4}\,dx = \int \frac{3}{x^2}\,dx + \int \frac{3}{4}\,dx.$

b. Long division simplifies the evaluation of the integral $\displaystyle\int \frac{x^3 + 2}{3x^4 + x}\,dx.$

c. $\displaystyle\int \frac{dx}{\sin x + 1} = \ln|\sin x + 1| + C.$

d. $\displaystyle\int \frac{dx}{e^x} = \ln e^x + C.$

42–54. Miscellaneous integrals *Use the approaches discussed in this section to evaluate the following integrals.*

42. $\displaystyle\int_4^9 \frac{dx}{1 - \sqrt{x}}$

43. $\displaystyle\int_{-1}^0 \frac{x}{x^2 + 2x + 2}\,dx$

44. $\displaystyle\int_0^1 \sqrt{1 + \sqrt{x}}\,dx$

45. $\displaystyle\int \sin x \sin 2x\,dx$

46. $\displaystyle\int_0^{\pi/2} \sqrt{1 + \cos 2x}\,dx$

47. $\displaystyle\int \frac{dx}{x^{1/2} + x^{3/2}}$

48. $\displaystyle\int_0^1 \frac{dp}{4 - \sqrt{p}}$

49. $\displaystyle\int \frac{x - 2}{x^2 + 6x + 13}\,dx$

50. $\displaystyle\int_0^{\pi/4} 3\sqrt{1 + \sin 2x}\,dx$

51. $\displaystyle\int \frac{e^x}{e^{2x} + 2e^x + 1}\,dx$

52. $\displaystyle\int_0^{\pi/8} \sqrt{1 - \cos 4x}\,dx$

53. $\displaystyle\int_1^3 \frac{2}{x^2 + 2x + 1}\,dx$

54. $\displaystyle\int_0^2 \frac{2}{s^3 + 3s^2 + 3s + 1}\,ds$

55. **Different substitutions**

a. Evaluate $\int \tan x \sec^2 x\,dx$ using the substitution $u = \tan x$.

b. Evaluate $\int \tan x \sec^2 x\,dx$ using the substitution $u = \sec x$.

c. Reconcile the results in parts (a) and (b).

56. **Different methods**

a. Evaluate $\int \cot x \csc^2 x\,dx$ using the substitution $u = \cot x$.

b. Evaluate $\int \cot x \csc^2 x\,dx$ using the substitution $u = \csc x$.

c. Reconcile the results in parts (a) and (b).

57. **Different methods**

a. Evaluate $\displaystyle\int \frac{x^2}{x + 1}\,dx$ using the substitution $u = x + 1$.

b. Evaluate $\displaystyle\int \frac{x^2}{x + 1}\,dx$ after first performing long division on the integrand.

c. Reconcile the results in parts (a) and (b).

58. **Different substitutions**

a. Show that $\displaystyle\int \frac{dx}{\sqrt{x - x^2}} = \sin^{-1}(2x - 1) + C$ using either $u = 2x - 1$ or $u = x - \frac{1}{2}$.

b. Show that $\displaystyle\int \frac{dx}{\sqrt{x - x^2}} = 2\sin^{-1}\sqrt{x} + C$ using $u = \sqrt{x}$.

c. Prove the identity $2\sin^{-1}\sqrt{x} - \sin^{-1}(2x - 1) = \frac{\pi}{2}$.

(*Source: The College Mathematics Journal* 32, 5, Nov 2001)

Applications

59. **Area of a region between curves** Find the area of the region bounded by the curves $y = \dfrac{x^2}{x^3 - 3x}$ and $y = \dfrac{1}{x^3 - 3x}$ on the interval $[2, 4]$.

60. **Area of a region between curves** Find the area of the entire region bounded by the curves $y = \dfrac{x^3}{x^2 + 1}$ and $y = \dfrac{8x}{x^2 + 1}$.

61. **Volumes of solids** Consider the region R bounded by the graph of $f(x) = \sqrt{x^2 + 1}$ and the x-axis on the interval $[0, 2]$.

a. Find the volume of the solid formed when R is revolved about the x-axis.

b. Find the volume of the solid formed when R is revolved about the y-axis.

62. **Volumes of solids** Consider the region R bounded by the graph of $f(x) = \dfrac{1}{x + 2}$ and the x-axis on the interval $[0, 3]$.

a. Find the volume of the solid formed when R is revolved about the x-axis.

b. Find the volume of the solid formed when R is revolved about the y-axis.

63. **Arc length** Find the length of the curve $y = x^{5/4}$ on the interval $[0, 1]$. (*Hint:* Write the arc length integral and let $u^2 = 1 + (\frac{5}{4})^2\sqrt{x}$.)

64. **Surface area** Find the area of the surface generated when the region bounded by the graph of $y = e^x + \frac{1}{4}e^{-x}$ on the interval $[0, \ln 2]$ is revolved about the x-axis.

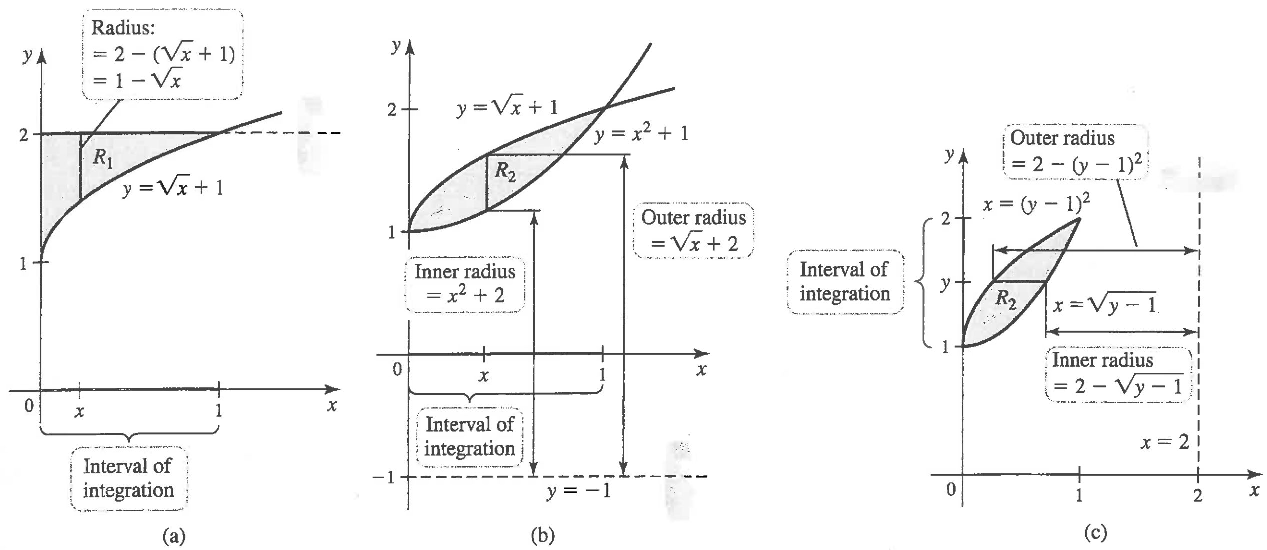


Figure 6.37

b. When the graph of f is revolved about $y = -1$, it sweeps out a solid of revolution whose radius at a point x is $f(x) + 1 = \sqrt{x} + 2$. Similarly, when the graph of g is revolved about $y = -1$, it sweeps out a solid of revolution whose radius at a point x is $g(x) + 1 = x^2 + 2$ (Figure 6.37b). Using the washer method, the volume of the solid generated when R_2 is revolved about $y = -1$ is

$$\int_0^1 \pi\left((\sqrt{x} + 2)^2 - (x^2 + 2)^2\right) dx$$

$$= \pi \int_0^1 (-x^4 - 4x^2 + x + 4\sqrt{x})\, dx$$

$$= \frac{49\pi}{30}.$$

c. When the region R_2 is revolved about the line $x = 2$, we use the washer method and integrate in the y-direction. First note that the graph of f is described by $y = \sqrt{x} + 1$, or equivalently, $x = (y - 1)^2$, for $y \geq 1$. Also, the graph of g is described by $y = x^2 + 1$, or equivalently, $x = \sqrt{y - 1}$ for $y \geq 1$ (Figure 6.37c). When the graph of f is revolved about the line $x = 2$, the radius of a typical disk at a point y is $2 - (y - 1)^2$. Similarly, when the graph of g is revolved about $x = 2$, the radius of a typical disk at a point y is $2 - \sqrt{y - 1}$. Finally, observe that the extent of the region R_2 in the y-direction is the interval $1 \leq y \leq 2$.

Applying the washer method, simplifying the integrand, and integrating powers of y, the volume of the solid of revolution is

$$\int_1^2 \pi\left((2 - (y - 1)^2)^2 - (2 - \sqrt{y - 1})^2\right) dy = \frac{31\pi}{30}.$$

Related Exercises 45–52 ◄

SECTION 6.3 EXERCISES

Review Questions

1. Suppose a cut is made through a solid object perpendicular to the x-axis at a particular point x. Explain the meaning of $A(x)$.

2. A solid has a circular base and cross sections perpendicular to the base are squares. What method should be used to find the volume of the solid?

3. The region bounded by the curves $y = 2x$ and $y = x^2$ is revolved about the x-axis. Give an integral for the volume of the solid that is generated.

4. The region bounded by the curves $y = 2x$ and $y = x^2$ is revolved about the y-axis. Give an integral for the volume of the solid that is generated.

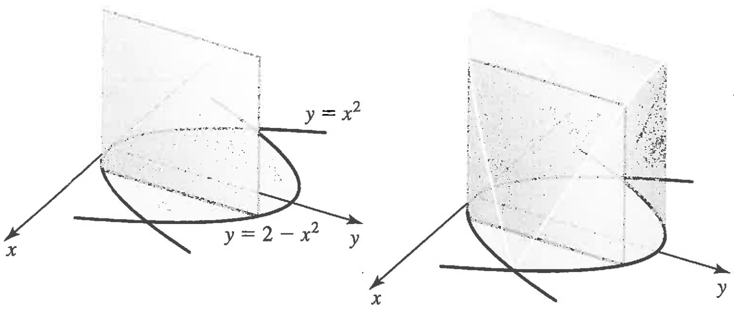
5. Why is the disk method a special case of the general slicing method?

6. The region R bounded by the graph of $y = f(x) \geq 0$ and the x-axis on $[a, b]$ is revolved about the line $y = -2$ to form a solid of revolution whose cross sections are washers. What are the inner and outer radii of the washer at a point x in $[a, b]$?

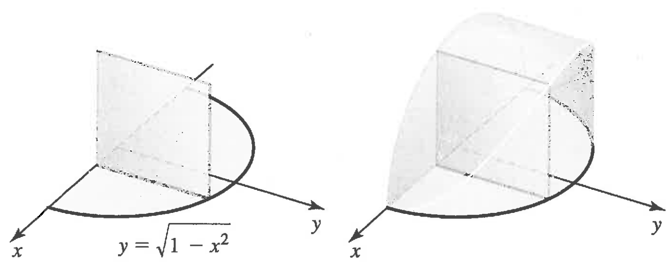
Basic Skills

7–16. General slicing method *Use the general slicing method to find the volume of the following solids.*

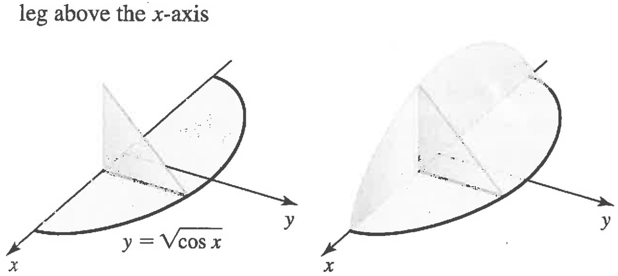
7. The solid whose base is the region bounded by the curves $y = x^2$ and $y = 2 - x^2$, and whose cross sections through the solid perpendicular to the x-axis are squares



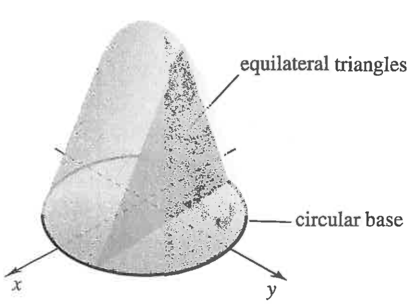
8. The solid whose base is the region bounded by the semicircle $y = \sqrt{1 - x^2}$ and the x-axis, and whose cross sections through the solid perpendicular to the x-axis are squares



9. The solid whose base is the region bounded by the curve $y = \sqrt{\cos x}$ and the x-axis on $[-\pi/2, \pi/2]$, and whose cross sections through the solid perpendicular to the x-axis are isosceles right triangles with a horizontal leg in the xy-plane and a vertical leg above the x-axis

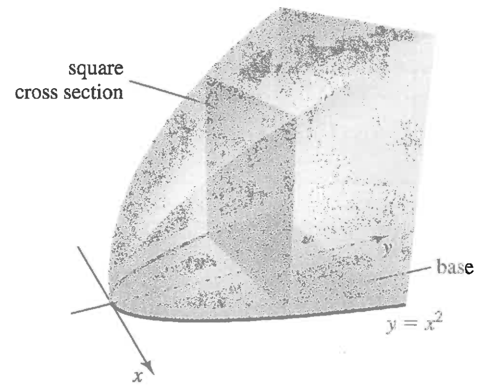


10. The solid with a circular base of radius 5 whose cross sections perpendicular to the base and parallel to the x-axis are equilateral triangles



11. The solid with a semicircular base of radius 5 whose cross sections perpendicular to the base and parallel to the diameter are squares

12. The solid whose base is the region bounded by $y = x^2$ and the line $y = 1$, and whose cross sections perpendicular to the base and parallel to the x-axis are squares

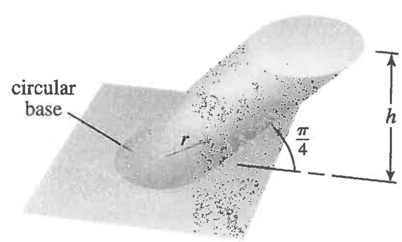


13. The solid whose base is the triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 2)$, and whose cross sections perpendicular to the base and parallel to the y-axis are semicircles

14. The pyramid with a square base 4 m on a side and a height of 2 m (Use calculus.)

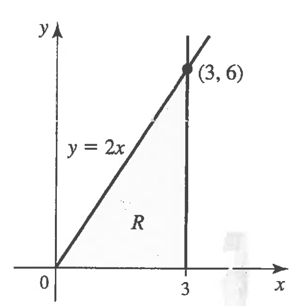
15. The tetrahedron (pyramid with four triangular faces), all of whose edges have length 4

16. A circular cylinder of radius r and height h whose axis is at an angle of $\pi/4$ to the base

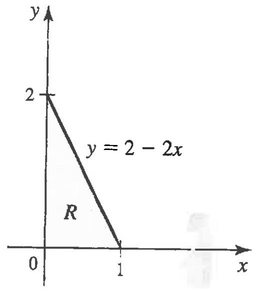


17–26. Disk method *Let R be the region bounded by the following curves. Use the disk method to find the volume of the solid generated when R is revolved about the x-axis.*

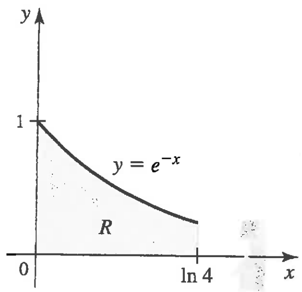
17. $y = 2x, y = 0, x = 3$ (Verify that your answer agrees with the volume formula for a cone.)



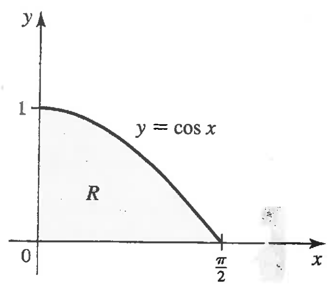
18. $y = 2 - 2x, y = 0, x = 0$ (Verify that your answer agrees with the volume formula for a cone.)



19. $y = e^{-x}, y = 0, x = 0, x = \ln 4$



20. $y = \cos x$ on $[0, \pi/2], y = 0, x = 0$ (Recall that $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$.)



21. $y = \sin x$ on $[0, \pi], y = 0$ (Recall that $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.)

22. $y = \sqrt{25 - x^2}, y = 0$ (Verify that your answer agrees with the volume formula for a sphere.)

23. $y = \dfrac{1}{\sqrt[4]{1 - x^2}}, y = 0, x = 0$, and $x = \frac{1}{2}$

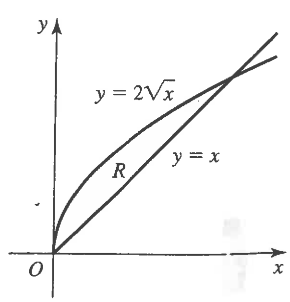
24. $y = \sec x, y = 0, x = 0$, and $x = \frac{\pi}{4}$

25. $y = \dfrac{1}{\sqrt{1 + x^2}}, y = 0, x = -1$, and $x = 1$

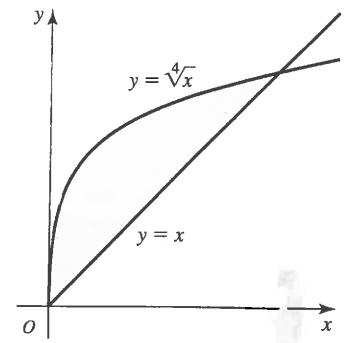
26. $y = \dfrac{1}{\sqrt[4]{1 - x^2}}, y = 0, x = -\frac{1}{2}$, and $x = \frac{1}{2}$

27–34. Washer method *Let R be the region bounded by the following curves. Use the washer method to find the volume of the solid generated when R is revolved about the x-axis.*

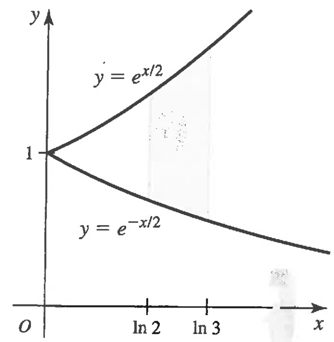
27. $y = x, y = 2\sqrt{x}$



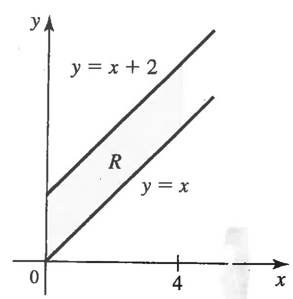
28. $y = x, y = \sqrt[4]{x}$



29. $y = e^{x/2}, y = e^{-x/2}, x = \ln 2, x = \ln 3$



30. $y = x, y = x + 2, x = 0, x = 4$



31. $y = x + 3, y = x^2 + 1$

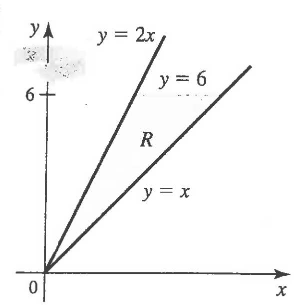
32. $y = \sqrt{\sin x}, y = 1, x = 0$

33. $y = \sin x, y = \sqrt{\sin x}$, for $0 \le x \le \pi/2$

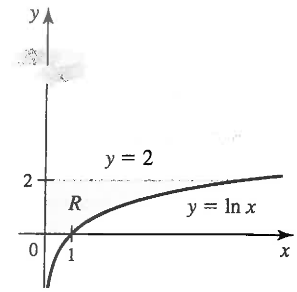
34. $y = |x|, y = 2 - x^2$

35–40. Disks / washers about the y-axis *Let R be the region bounded by the following curves. Use the disk or washer method to find the volume of the solid generated when R is revolved about the y-axis.*

35. $y = x, y = 2x, y = 6$



36. $y = 0, y = \ln x, y = 2, x = 0$



37. $y = x^3, y = 0, x = 2$

38. $y = \sqrt{x}, y = 0, x = 4$

39. $x = \sqrt{4 - y^2}, x = 0$

40. $y = \sin^{-1} x, x = 0, y = \pi/4$

41–44. Which is greater? *For the following regions R, determine which is greater—the volume of the solid generated when R is revolved about the x-axis or about the y-axis.*

41. R is bounded by $y = 2x$, the x-axis, and $x = 5$.

42. R is bounded by $y = 4 - 2x$, the x-axis, and the y-axis.

43. R is bounded by $y = 1 - x^3$, the x-axis, and the y-axis.

44. R is bounded by $y = x^2$ and $y = \sqrt{8x}$.

45–52. Revolution about other axes *Find the volume of the solid generated in the following situations.*

45. The region R bounded by the graphs of $x = 0, y = \sqrt{x}$, and $y = 1$ is revolved around the line $y = 1$.

46. The region R bounded by the graphs of $x = 0, y = \sqrt{x}$, and $y = 2$ is revolved around the line $x = 4$.

47. The region R bounded by the graph of $y = 2 \sin x$ and the x-axis on $[0, \pi]$ is revolved about the line $y = -2$.

48. The region R bounded by the graph of $y = \ln x$ and the y-axis on the interval $0 \le y \le 1$ is revolved about the line $x = -1$.

49. The region R bounded by the graphs of $y = \sin x$ and $y = 1 - \sin x$ on $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ is revolved about the line $y = -1$.

50. The region R in the first quadrant bounded by the graphs of $y = x$ and $y = 1 + \frac{x}{2}$ is revolved about the line $y = 3$.

51. The region R in the first quadrant bounded by the graphs of $y = 2 - x$ and $y = 2 - 2x$ is revolved about the line $x = 3$.

52. The region R is bounded by the graph of $f(x) = 2x(2 - x)$ and the x-axis. Which is greater, the volume of the solid generated when R is revolved about the line $y = 2$ or the volume of the solid generated when R is revolved about the line $y = 0$? Use integration to justify your answer.

Further Explorations

53. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. A pyramid is a solid of revolution.

b. The volume of a hemisphere can be computed using the disk method.

c. Let R_1 be the region bounded by $y = \cos x$ and the x-axis on $[-\pi/2, \pi/2]$. Let R_2 be the region bounded by $y = \sin x$ and the x-axis on $[0, \pi]$. The volumes of the solids generated when R_1 and R_2 are revolved about the x-axis are equal.

54–60. Solids of revolution *Find the volume of the solid of revolution. Sketch the region in question.*

54. The region bounded by $y = (\ln x)/\sqrt{x}, y = 0$, and $x = 2$ revolved about the x-axis

55. The region bounded by $y = 1/\sqrt{x}, y = 0, x = 2$, and $x = 6$ revolved about the x-axis

56. The region bounded by $y = \dfrac{1}{\sqrt{x^2 + 1}}$ and $y = \dfrac{1}{\sqrt{2}}$ revolved about the x-axis

57. The region bounded by $y = e^x, y = 0, x = 0$, and $x = 2$ revolved about the x-axis

58. The region bounded by $y = e^{-x}, y = e^x, x = 0$, and $x = \ln 4$ revolved about the x-axis

59. The region bounded by $y = \ln x, y = \ln x^2$, and $y = \ln 8$ revolved about the y-axis

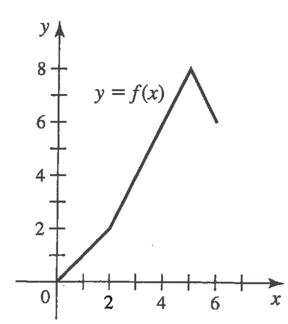
60. The region bounded by $y = e^{-x}, y = 0, x = 0$, and $x = p > 0$ revolved about the x-axis (Is the volume bounded as $p \to \infty$?)

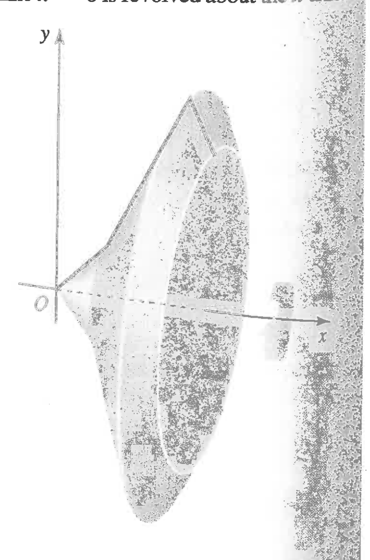
61. Fermat's volume calculation (1636) Let R be the region bounded by the curve $y = \sqrt{x + a}$ (with $a > 0$), the y-axis, and the x-axis. Let S be the solid generated by rotating R about the y-axis. Let T be the inscribed cone that has the same circular base as S and height $\sqrt{a}$. Show that volume(S)/volume$(T) = \frac{8}{5}$.

62. Solid from a piecewise function Let

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 2 \\ 2x - 2 & \text{if } 2 < x \le 5 \\ -2x + 18 & \text{if } 5 < x \le 6. \end{cases}$$

Find the volume of the solid formed when the region bounded by the graph of f, the x-axis, and the line $x = 6$ is revolved about the x-axis.





SECTION 6.4 EXERCISES

Review Questions

1. Assume f and g are continuous with $f(x) \geq g(x)$ on $[a, b]$. The region bounded by the graphs of f and g and the lines $x = a$ and $x = b$ is revolved about the y-axis. Write the integral given by the shell method that equals the volume of the resulting solid.

2. Fill in the blanks: A region R is revolved about the y-axis. The volume of the resulting solid could (in principle) be found using the disk/washer method and integrating with respect to ________ or using the shell method and integrating with respect to ________.

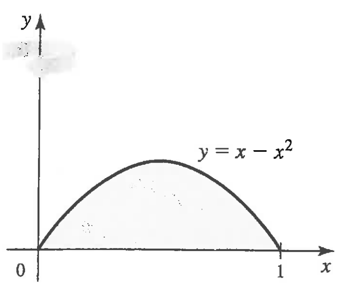
3. Fill in the blanks: A region R is revolved about the x-axis. The volume of the resulting solid could (in principle) be found using the disk/washer method and integrating with respect to ________ or using the shell method and integrating with respect to ________.

4. Are shell method integrals easier to evaluate than washer method integrals? Explain.

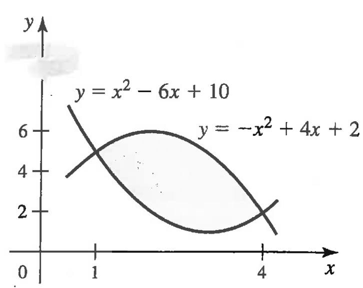
Basic Skills

5–14. Shell method *Let R be the region bounded by the following curves. Use the shell method to find the volume of the solid generated when R is revolved about the y-axis.*

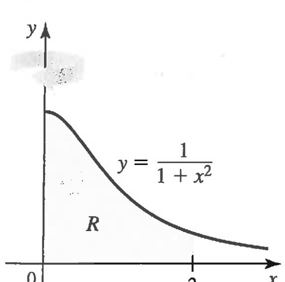
5. $y = x - x^2, y = 0$



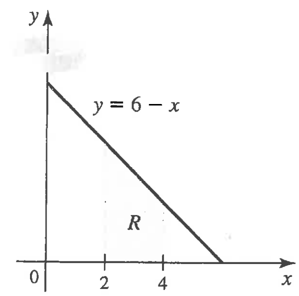
6. $y = -x^2 + 4x + 2, y = x^2 - 6x + 10$



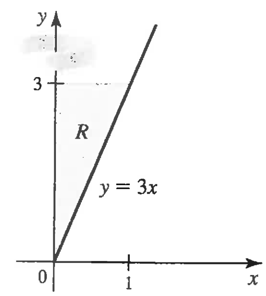
7. $y = (1 + x^2)^{-1}, y = 0, x = 0$, and $x = 2$



8. $y = 6 - x, y = 0, x = 2$, and $x = 4$



9. $y = 3x, y = 3$, and $x = 0$ (Use integration and check your answer using the volume formula for a cone.)



10. $y = 1 - x^2, x = 0$, and $y = 0$, in the first quadrant

11. $y = x^3 - x^8 + 1, y = 1$

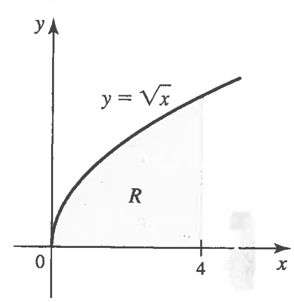
12. $y = \sqrt{x}, y = 0$, and $x = 1$

13. $y = \cos x^2, y = 0$, for $0 \leq x \leq \sqrt{\pi/2}$

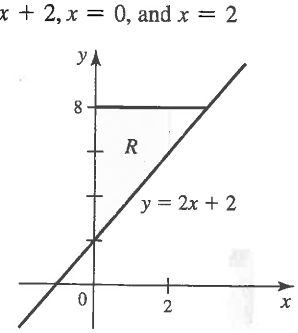
14. $y = \sqrt{4 - 2x^2}, y = 0$, and $x = 0$, in the first quadrant

15–26. Shell method *Let R be the region bounded by the following curves. Use the shell method to find the volume of the solid generated when R is revolved about the x-axis.*

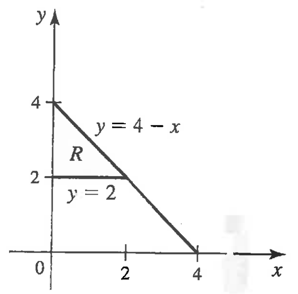
15. $y = \sqrt{x}, y = 0$, and $x = 4$



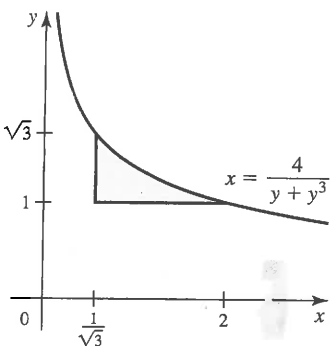
16. $y = 8, y = 2x + 2, x = 0$, and $x = 2$



17. $y = 4 - x$, $y = 2$, and $x = 0$



18. $x = \dfrac{4}{y + y^3}$, $x = \dfrac{1}{\sqrt{3}}$, and $y = 1$



19. $y = x$, $y = 2 - x$, and $y = 0$

20. $x = y^2$, $x = 4$, and $y = 0$

21. $x = y^2$, $x = 0$, and $y = 3$

22. $y = x^3$, $y = 1$, and $x = 0$

23. $y = 2x^{-3/2}$, $y = 2$, $y = 16$, and $x = 0$

24. $y = \sqrt{\sin^{-1} x}$, $y = \sqrt{\pi/2}$, and $x = 0$

25. $y = \sqrt{\cos^{-1} x}$, in the first quadrant

26. $y = \sqrt{50 - 2x^2}$, in the first quadrant

27–32. Shell method *Use the shell method to find the volume of the following solids.*

27. A right circular cone of radius 3 and height 8

28. The solid formed when a hole of radius 2 is drilled symmetrically along the axis of a right circular cylinder of height 6 and radius 4

29. The solid formed when a hole of radius 3 is drilled symmetrically along the axis of a right circular cone of radius 6 and height 9

30. The solid formed when a hole of radius 3 is drilled symmetrically through the center of a sphere of radius 6

31. The *ellipsoid* formed when that part of the ellipse $x^2 + 2y^2 = 4$ with $x \geq 0$ is revolved about the y-axis

32. A hole of radius $r \leq R$ is drilled symmetrically along the axis of a bullet. The bullet is formed by revolving the parabola $y = 6\left(1 - \dfrac{x^2}{R^2}\right)$ about the y-axis, where $0 \leq x \leq R$.

33–36. Shell method about other lines *Let R be the region bounded by $y = x^2$, $x = 1$, and $y = 0$. Use the shell method to find the volume of the solid generated when R is revolved about the following lines.*

33. $x = -2$
34. $x = 1$
35. $y = -2$
36. $y = 2$

37–40. Different axes of revolution *Use either the washer or shell method to find the volume of the solid that is generated when the region in the first quadrant bounded by $y = x^2$, $y = 1$, and $x = 0$ is revolved about the following lines.*

37. $y = -2$
38. $x = -1$
39. $y = 6$
40. $x = 2$

41–48. Washers vs. shells *Let R be the region bounded by the following curves. Let S be the solid generated when R is revolved about the given axis. If possible, find the volume of S by both the disk/washer and shell methods. Check that your results agree and state which method is easier to apply.*

41. $y = x$, $y = x^{1/3}$ in the first quadrant; revolved about the x-axis

42. $y = x^2$, $y = 2 - x$, and $x = 0$ in the first quadrant; revolved about the y-axis

43. $y = 1/(x + 1)$, $y = 1 - x/3$; revolved about the x-axis

44. $y = (x - 2)^3 - 2$, $x = 0$, and $y = 25$; revolved about the y-axis

45. $y = \sqrt{\ln x}$, $y = \sqrt{\ln x^2}$, and $y = 1$; revolved about the x-axis

46. $y = 6/(x + 3)$, $y = 2 - x$; revolved about the x-axis

47. $y = x - x^4$, $y = 0$; revolved about the x-axis

48. $y = x - x^4$, $y = 0$; revolved about the y-axis

Further Explorations

49. **Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample.
   a. When using the shell method, the axis of the cylindrical shells is parallel to the axis of revolution.
   b. If a region is revolved about the y-axis, then the shell method must be used.
   c. If a region is revolved about the x-axis, then in principle, it is possible to use the disk/washer method and integrate with respect to x or the shell method and integrate with respect to y.

50–54. Solids of revolution *Find the volume of the following solids of revolution. Sketch the region in question.*

50. The region bounded by $y = (\ln x)/x^2$, $y = 0$, and $x = 3$ revolved about the y-axis

51. The region bounded by $y = 1/x^2$, $y = 0$, $x = 2$, and $x = 8$ revolved about the y-axis

52. The region bounded by $y = 1/(x^2 + 1)$, $y = 0$, $x = 1$, and $x = 4$ revolved about the y-axis

53. The region bounded by $y = e^x/x$, $y = 0$, $x = 1$, and $x = 2$ revolved about the y-axis

54. The region bounded by $y^2 = \ln x$, $y^2 = \ln x^3$, and $y = 2$ revolved about the x-axis

55–62. Choose your method *Find the volume of the following solids using the method of your choice.*

55. The solid formed when the region bounded by $y = x^2$ and $y = 2 - x^2$ is revolved about the x-axis

56. The solid formed when the region bounded by $y = \sin x$ and $y = 1 - \sin x$ between $x = \pi/6$ and $x = 5\pi/6$ is revolved about the x-axis

57. The solid formed when the region bounded by $y = x$, $y = 2x + 2$, $x = 2$, and $x = 6$ is revolved about the y-axis